LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2024**

PMT3ME02 – ACTUARIAL MATHEMATICS

Date: 19-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)	
	Answer ALL the questions	**(5 x 1 = 5)**
1	**Answer the following**	
a)	Define survival analysis in life table.	
b)	Explain life expectancy.	
c)	Describe Combined insurance-annuity benefits.	
d)	Explain temporary life annuity with an example.	
e)	What do you mean by deferred annuities with annual premiums?	
	SECTION A – K2 (CO1)	
	Answer ALL the questions	**(5 x 1 = 5)**
2	**MCQ**	
a)	A basic device to estimate the mortality pattern is (i) standard table (ii) life table (iii) survival table (iv) discount table.	
b)	The expected present value of the Gross Premiums is (i) Expected present value of the net premium + Expected Present value of the Expense (ii) Expected present value of the net premium - Expected Present value of the Expense (iii) Expected present value of the net premium * Expected Present value of the Expense (iv)Expected Present value of the Expense - Expected present value of the net premium	
c)	An insurance company offers a 10% discount on insurance for a second bike to the customers who already have bike insurance policies with it. This is an example of: (i) first-degree price discrimination. (ii) second-degree price discrimination. (iii) third-degree price discrimination. (iv) peak-load pricing.	
d)	Life insurance benefits and life annuity benefits in the same contract then the policy is known as (i) endowment insurance (ii) premium (iii) insurance policy (iv) none of the above	
e)	An insurance provides a payment at some future time n if (x) is alive, and in addition a death benefit if (x) dies before time n then it is said to be (i)endowment insurance (ii)whole life insurance (iii)policy investment (iv)net annual premium	

SECTION B – K3 (CO2)

Answer any THREE of the following **(3 x 10 = 30)**

3 Defend the following statement $\dot{e}_x$ ¿ $\frac{1}{2}q_x + px\left(1+\dot{e}_{x+1}\right)$.

4 Determine the present value of an annuity, immediate of amount 100 payable quarterly for 10 years at the annual rate of interest of 8% convertible quarterly. Also produce its future value at the end of 10 years.

5 Calculate $\ddot{a}60(14), \ddot{a}[59]+1(14), \ddot{a}[58]+2(14)$ from the given data $q_{60} = 0.05$, $q_{61} = 0.10$, $q_{62} = 0.15$, $i0 = i_1 = 0.04$ and $i_k = 0.08$ for $k \geq 2$.

6 A company wants to provide a retirement plan for an employee who is aged 55 now. The plan will provide her with an annuity-immediate of $7,000 every year for 15 years upon her retirement at the age of 65. The company is funding this plan with an annuity- due of 10 years. If the rate of interest is 5%, evaluate the amount of installment the company should pay.

7 A temporary insurance is taken out by a person aged 40. The insurance pays a benefit of $2000 at the end of the year of death if death occurs before the age of 60. Using a 6% annual rate of interest, find the present value of the insurance. Also, find the standard deviation of the insurance.

SECTION C – K4 (CO3)

Answer any TWO of the following **(2 x 12.5 = 25)**

8 Given an extract from a life table.

x	l_x	d_x
30	10 000.00	34.78
31	9 965.22	38.10
32	9 927.12	41.76
33	9 885.35	45.81
34	9 839.55	50.26
35	9 789.29	55.17
36	9 734.12	60.56
37	9 673.56	66.49
38	9 607.07	72.99
39	9 534.08	80.11

Enumerate: (a) l_{40} (b) ${}_{10}p_{30}$, (c) q_{35}, (d) ${}_5q_{30}$, and (e) the probability that a life currently aged exactly 30 dies between ages 35 and 36.

9 A temporary life annuity on (50) provides for one payable at age 50, two payable at age 51, three payable at age 52 and four payable at age 53. Suppose $q_{50}=0.1$, $q_{51}=0.2$, $q_{52}=0.25$. The interested rate is 50% for the first year and 100% thereafter. Estimate the single premium

10 Given that $\ddot{a}40 = 15, \ddot{a}[40]+25 = 10$, $v(25) = 0.5$, $25p40 = 0.4$, find the net annual premium, payable for 25 years beginning at age 40, for a deferred annuity, paying 1000 yearly for life, with the first benefit payment at age 65.

11 Explain types of life insurance and procedure to calculate premiums.

SECTION D – K5 (CO4)

Answer any ONE of the following **(1 x 15 = 15)**

12 (a) Given that q_{70} = 0.010413 and q_{71} = 0.011670, calculate ${}_{0.7}q_{70.6}$ assuming uniform distribution of deaths.

(b) Suppose that $l_x = 100 - x$, $x = 0,1,\ldots,100$. Analyze the expressions of (a) ${}_np_x$, (b) ${}_nq_x$, (c) the probability that (x) will die between the ages of $x+n$ and $x+n+k$. (7+8 marks)

13	For a certain insurance contract, on (50), the death benefit for the first year of the contract is 1100, payable at the end of the year of death. The single premium for the whole contract is 600. This is based on an interest rate of 10% for the first year and a mortality table with q_{50} = 0.20. If the value of q_{50} is changed to 0.25, while all other value of q_x are unchanged, criticize the new single premium.

	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	(a) Suppose that out of a typical group of 100 people age 70, 10 will die in the first year, 15 will die in the second year, and 20 will die in the third year. Compile q_{70}, q_{71}, q_{72} and ${}_3p_{70}$. (b) A life annuity contract on (80) has a present value of 3.14. The annuity benefits at both age 80 and 81 are 1. The interest rate in the first year is 25% and p_{80}= 0.8. Suppose that the value of p_{81} increases by 10% while all other mortality rates remain unchanged. Evaluate the new present value of the contract? (10+10 marks)
15	(a) A certain electrical appliance is sold with a 5-year guarantee. This provides that the full purchase price is refunded if the product fails within 2 years, and half of the purchase price is refunded if the product fails in the following 3 years. A study shows that out of a typical batch of 100 items, there will be 2 failures in the first year, 3 failures in the second year and 4 failures per year after that. Assuming that interest is a constant 5% and that reimbursement is made at the end of the year of failure, what is the cost of this guarantee to the manufacturer, as a percentage of the purchase price? (b) Construct the relationship between y_x for various values of x. (10+10 marks)

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